Learning Objective

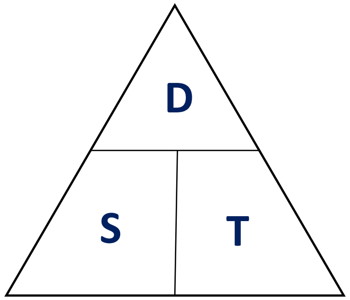
I will be able to determine the average rate of change using functions.

Success Criteria

At the end of the lesson, I will be able to:

- **calculate the average rate of change**

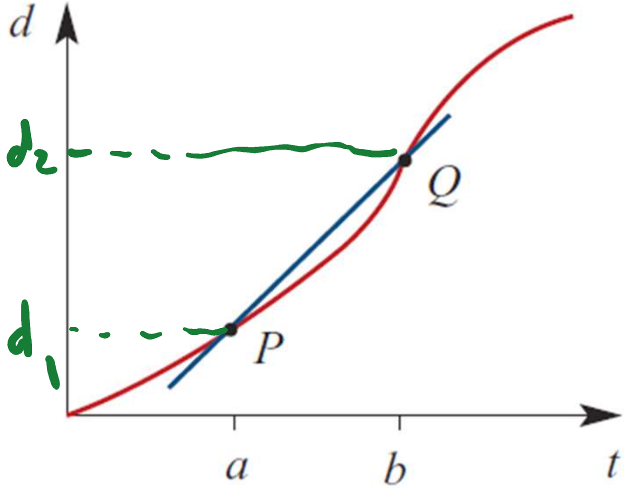
Average Speed



$$Average\ Speed = \frac{Total\ distance\ travelled}{Total\ time\ taken}$$

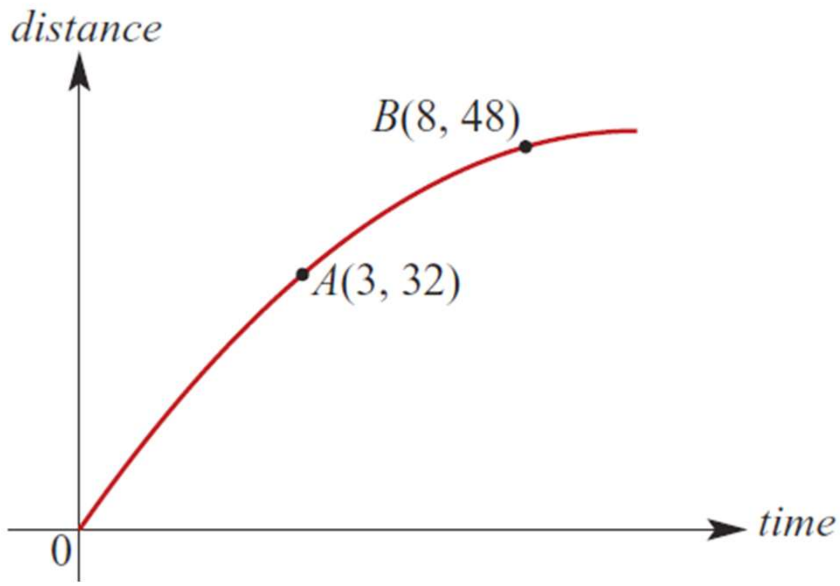
To find the average speed for any time interval $a \leq t \leq b$
We take the distance travelled between the time interval,
divide by the distance travelled.

$$\therefore \frac{d_2 - d_1}{b - a}$$



Guided Practice

The graph of distance travelled (metres) against time (seconds) for the motion of an object is shown. Find the average speed of the object in m/s over the interval t=3 to t=8.



Average speed = $\frac{48-32}{8-3}$

Average speed = $\frac{16}{5} = 3.2\ m/s$

Concept Development

Average rate of change for a function

Secant : A line that passes through two points on a curve

Chord: A line that joins two points on a curve

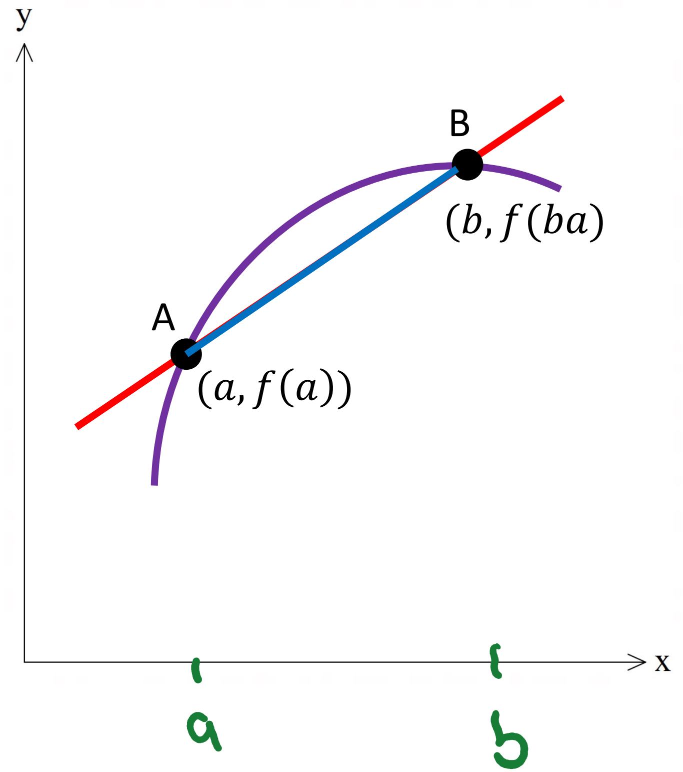
For any function $y = f(x)$ which passes through the interval $a \leq x \leq b$

Coordinates of A = $(a, f(a))$

x - input   output

Coordinates of B = $(b, f(b))$

Rate of change= $\frac{f(b)-f(a)}{b-a}$

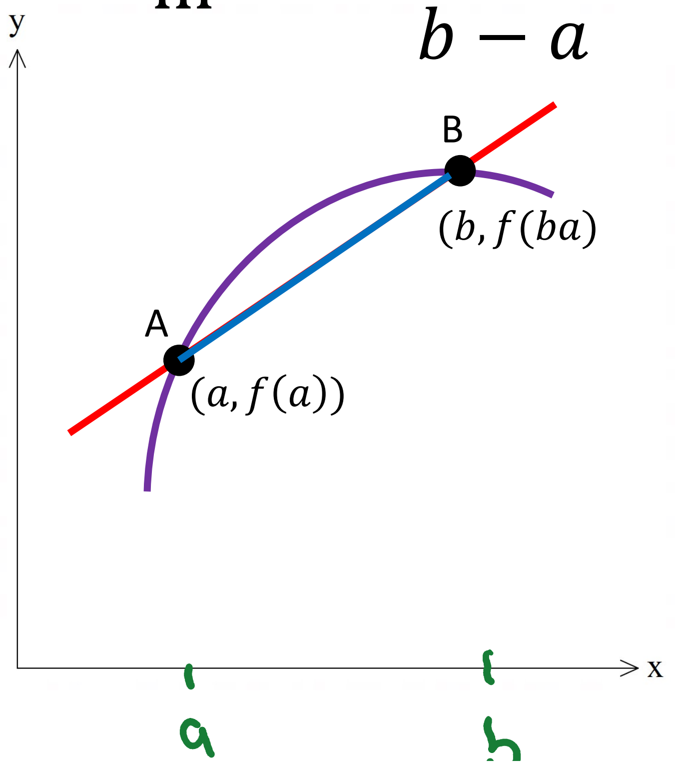


Rate of Change in ClassPad E-Activity

Given a function $y = f(x)$,

Average rate of change,

$$m = \frac{f(b) - f(a)}{b - a}$$



Guided Practice

Find the average of change of the function with rule $f(x) = x^2 - 2x + 5$ as x changes from 1 to 5.

Rate of change= $\frac{f(b)-f(a)}{b-a}$

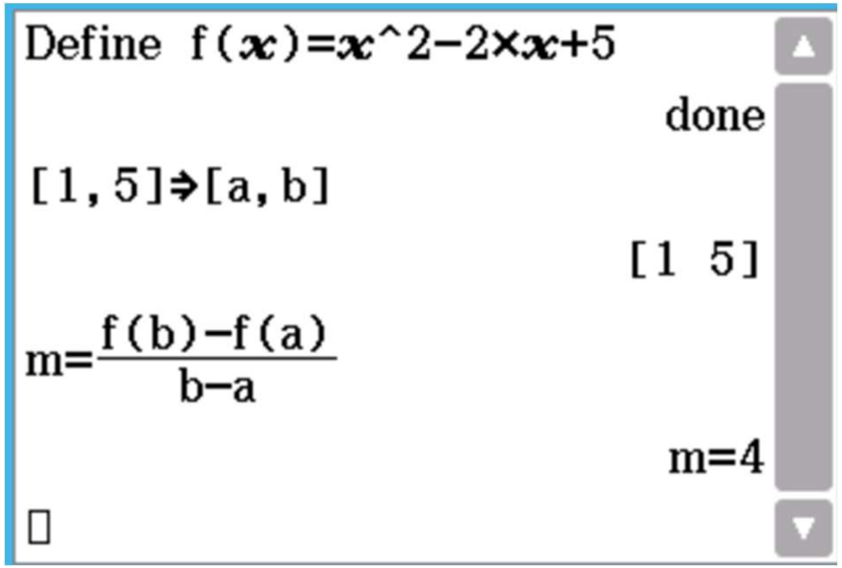
$$f(1) = (1)^2 - 2(1) + 5 = 4$$

$$f(5) = (5)^2 - 2(5) + 5 = 20$$

Average rate of change= $\frac{f(5)-f(1)}{5-1}$

Average rate of change= $\frac{20-4}{4}$

Average rate of change= 4



Guided Practice

Find the average of change of the function with rule $f(x) = \sqrt{5-x}$ as x changes from 0 to 4.

Rate of change$= \frac{f(b)-f(a)}{b-a}$

$f(0) = \sqrt{5}$

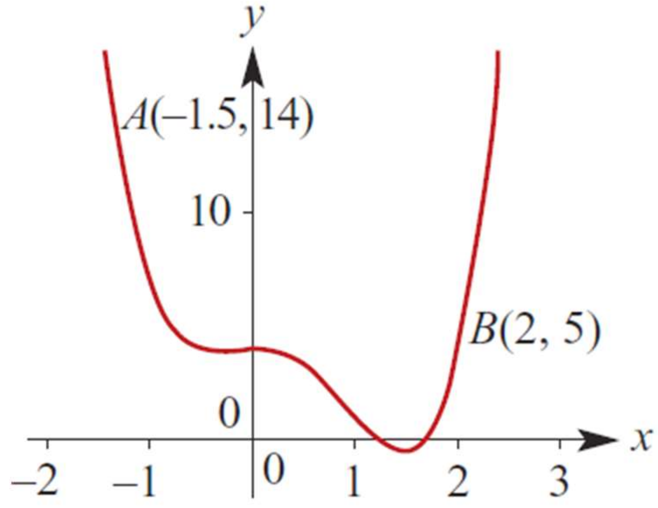
$f(4) = 1$

Average rate of change$= \frac{f(4)-f(0)}{4-0}$

Average rate of change$= \frac{1-\sqrt{5}}{4}$

Guided Practice

Find the average rate of change from point A to B.



Average speed = $\frac{5-14}{2-(1.5)}$

Average speed = $\frac{-9}{3.5} = -\frac{18}{7}$

Guided Practice

A person invests $2000 dollars, which increases in value by 7% per year for three years.

a) Calculate the value of the investment after three years.

b) Calculate the average rate of change in the value of the investment over that time.

Value of investment after first year$= 2000 \times 1.07 = \$2140$

Value of investment after second year$= 2140 \times 1.07 = \$2289.80$

Value of investment after third year$= 2289.80 \times 1.07 = \$2450.09$

$$\text{Average rate of change} = \frac{2450.09-2000}{3-0} = \$150.03 \text{ per year.}$$

Guided Practice

The depth, d cm, of water in a bath tub t minutes after the tap is turned on is modelled by the function $d(t) = -\frac{300}{t+6} + 50, t \geq 0$. Find the average rate of change of the depth of the water with respect to time over the first 10 minutes after the tap is turned on.

$$\text{At t} = 0, d(0) = -\frac{300}{0+6} + 50 = 0$$

$$\text{At t} = 10, d(10) = -\frac{300}{16} + 50 = 31.25$$

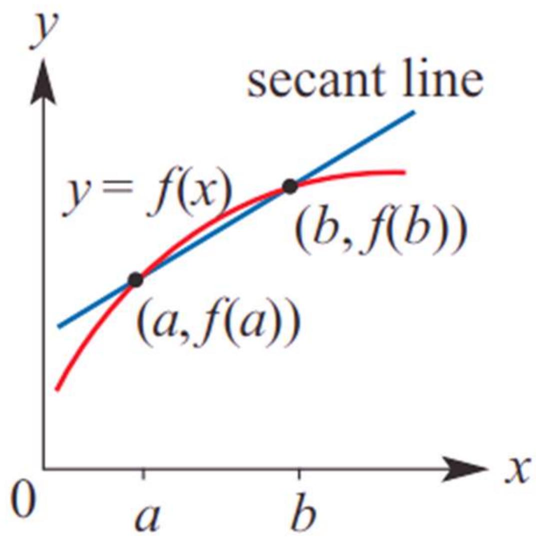
$$\text{Rate of change} = \frac{31.25-0}{10-0} = 3.125 \text{ cm/minutes}$$

In Summary

- The line which passes through two points on a curve is called a **secant**.
- The line segment joining two points on a curve is called a **chord**.
- For a function $y = f(x)$, the **average rate of change** of y with respect to x over the interval $[a, b]$ is the gradient of the secant line through $(a, f(a))$ and $(b, f(b))$.

  That is,

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$



Independent Practice

Complete Cambridge Ex 17A